

Simulating Decorative Mosaics

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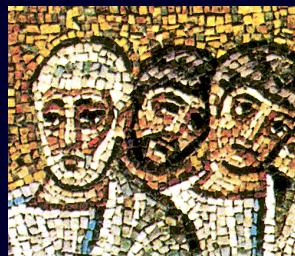
Mosaic Tile Simulation

- Reproduce mosaic tilings
 - long-lasting (graphics is ephemeral)
 - realism with very few pixels
 - pixels have orientation

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Real Tile Mosaics



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Real Mosaics II



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The Problem

- Square tiles cover the plane perfectly
- Variable orientation → loose packing
- Opposing goals:
 - non-uniform grid
 - dense packing

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Previous Work

- Romans: algorithm?
- Relaxation (Haeberli 90)
 - scatter points on image
 - voronoi region = tile
 - tiles not square

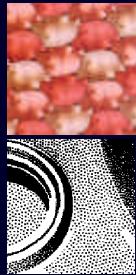


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Previous Work

- Escherization (Kaplan 00)
 - regular tilings
 - use symmetry groups
- Stippling (Deussen 01)
 - voronoi relaxation
 - round dots



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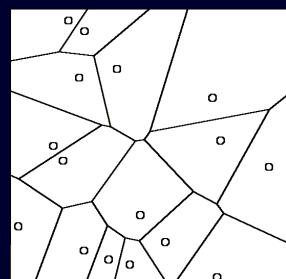
Voronoi Diagram

- What is it?
 - Input: sites (generators)
 - Result: regions closest to each site
- How to compute it?
 - Divide & conquer (Preparata)
 - sweep-line (Fortune)
 - incremental

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Voronoi Diagram



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Centroidal Voronoi Diagrams

- VD sites $\stackrel{1}{\sim}$ centroids
- Lloyd's alg. (k-means):
 - 1: move site to centroid
 - 2: recalculate VD
 - 3: repeat

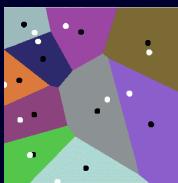


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Centroidal Voronoi Diagrams

- VD sites $\stackrel{1}{\sim}$ centroids
- Lloyd's alg. (k-means):
 - 1: **move site to centroid**
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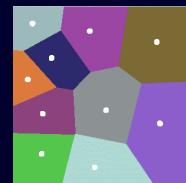


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Centroidal Voronoi Diagrams

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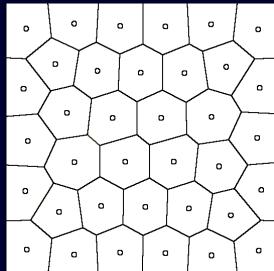


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CVD After Convergence

Almost a **hexagonal** grid



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CVD properties

- Minimum-energy arrangements
- In Nature:
 - honeycombs
 - giraffe spots
- Point Sampling:
 - approximates Poisson-disk (low discrepancy)
 - can bias for filter function

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Key Idea 1

- CVDs and Tilings
 - best circle packing = hexagonal tiling
 - Euclidean metric: CVD = "hexagonal" tiling
 - different metric: CVD = ?
- Mosaics are Tilings
 - non-periodic
 - locally-varying orientation

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Key Idea 1_(continued)

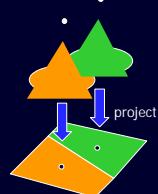
- Create "mosaic" metric
 - locally Manhattan (L_1)
 - locally varying orientation
- BUT:
 - existing algorithms limited
 - they assume Euclidean metric
 - if L_1 metric: they assume isotropic

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Hardware-assisted VD's

- Hoff 99
 - uses graphics hardware
 - draw cone at each site
 - orthogonal view from above
 - region color = cone color
 - can extend to non-point sites (curves)

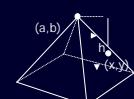
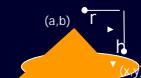


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Key idea 2

- Cone is distance function
 - radius = height
- Non-euclidean distance: $h^2 = (x-a)^2 + (y-b)^2$
 - different kind of cone
 - eg square pyramid
 - can be non-isotropic (rotate pyramid on Z axis)



$$h = |x-a| + |y-b|$$

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Basic Tiling Algorithm

- Compute orientation field ([details later](#))
- initialize: random points on image
 - use pyramids to get oriented tiles
- use Lloyd's method
 - spreads sites evenly
- draw oriented tile at each site

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Details

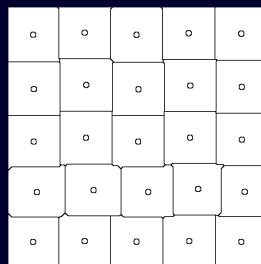
- Lloyd's method:
 - to get Voronoi region centroids:
 - 1: read back pixels (frame buffer)
 - 2: get average (row,col) [per colour](#)
 - 3: convert back to object coords.
 - move sites to centroids
 - repeat until converged

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Lloyd Near Convergence

Manhattan metric
(isotropic)



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Tile Orientations



- Artisan's algorithm
 - draw region boundaries
 - line tiles up on boundaries
 - build concentric rows
- Tile orientations
 - emphasize boundary curves
 - near curve: must follow curve
 - far from curve: don't care

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Orientation Field

- Vector Field
 - $\varphi(x,y)$ unit vector at each (x,y)
 - near a curve: $\varphi = \text{curve direction}$
 - far away: less curve influence
- Align tiles to φ
 - rotate Manhattan-metric pyramids
 - rotate square tiles

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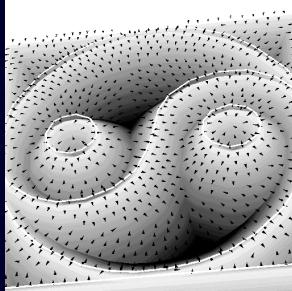
Computing $\varphi(x,y)$

- Choose curves that need emphasis
- get VD for curves (Hoff 99)
 - draw generalized "cones"
- $z(x,y) = \text{eye distance} = z\text{-buffer}(x,y)$
 - $z = \text{distance from curve}$
- get gradient, normalize
 - $\varphi(x,y) = \nabla z \div |\nabla z|$

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Generalized Cones



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Edge Discrimination

- Problem: Tiles on Curve
 - ϕ same on both sides ($\text{mod } 180^\circ$)
 - rotate tile $180^\circ \rightarrow$ no change
 - Lloyd's alg. ignores edges
 - result: tiles straddle curves

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Edge Discrimination

- Solution: use hardware
 - draw edge thick, different color
 - edge overwrites part of tile
 - centroids move away from edge
 - apply Lloyd's alg.
 - leaves gap where edge was.

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Edge Discrimination

- **Tiles on edge**
 - new centroids
 - move sites
 - repeat Lloyd's
 - gap created
 - repeat Lloyd's
 - edge is clear

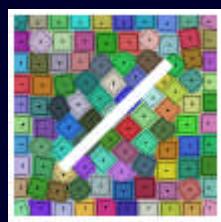


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Edge Discrimination

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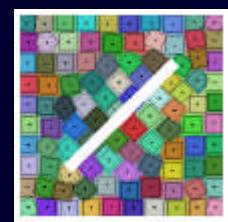


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Edge Discrimination

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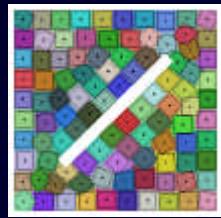


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Edge Discrimination

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Edge Discrimination

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Edge Discrimination

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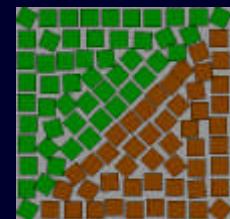


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Edge Discrimination

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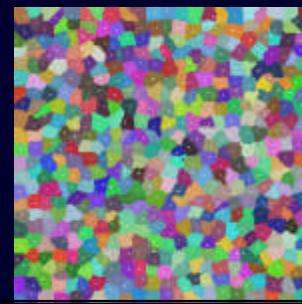
Putting It All Together

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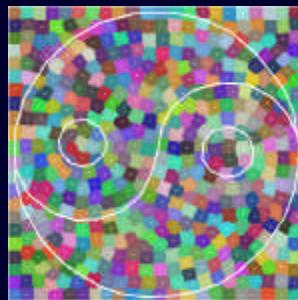
Putting It All Together

Initial random
tiles



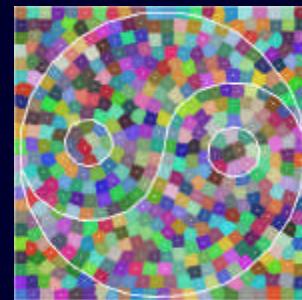
Putting It All Together

20 iterations
of Lloyd's



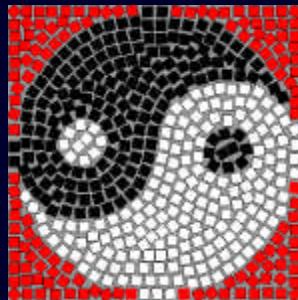
Putting It All Together

Edges cleared



Putting It All Together

Final tiling



Tile Attributes

- We have:
 - tile position, orientation
- Can also control:
 - size
 - aspect ratio
 - shape (round, square, diamond)
 - colour

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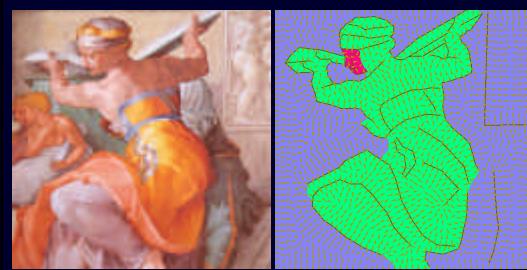
Variable-Size Tiles

- Scaled cones
 - $h = \alpha (|x-a| + |y-b|)$
- Initial tile positions?
 - uniform → slow convergence
 - rejection sampling

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Lybian Sibyl



Original

Curves, regions, $\phi(x,y)$

Size Variation



2000 **variable**-size tiles

2000 **equal**-size tiles

Tiles Carry More Information



2000 TILES

2000 PIXELS

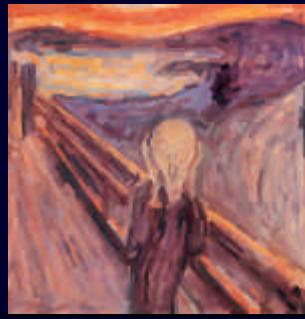
Other Effects: "Painterly"



Oval over-size
overlapping tiles

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Other Effects: Regions



Voronoi regions
with color
from image

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More Pictures



"Second Story Sunlight" (Hopper 60)

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More Pictures



Stained-glass
photograph

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More Pictures



Photograph:
Seal on Beach

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Summary

- Pack tiles on curvilinear grid
 - Low-energy particle arrangements
- Generalized CVD
 - Voronoi diagram for any metric
 - Use graphics hardware

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Further Work

- Reduce "grout"
 - final pass: adjust tile shapes
 - currently don't use adjacency info
- Paint strokes
 - textured tiles

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Further Work

- User-defined φ
 - better tile orientations
- Tile shapes
 - higher moments: Σx^2 , Σxy , etc.
- Dithering
 - no regular grid

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